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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (REVISED)

Term-End Examination

June, 2023

BCS-012 : BASIC MATHEMATICS

Time : 3 Hours *Maximum Marks : 100*

Note : *Question Number **1** is compulsory. Attempt any **three** questions from the remaining questions.*

1. (a) If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, find a and b. 5

(b) Show that $n(n+1)(2n+1)$ is a multiple of 6 for every natural number n. 5

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(c) If 1, ω and ω^2 are cube roots of unity, show that : 5

$$(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49.$$

(d) Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$. 5

(e) Solve the equation $2x^3 - 15x^2 + 37x - 30 = 0$, given that the roots of the equation are in A.P. 5

(f) Evaluate the integral : 5

$$I = \int \frac{x^2}{(x+1)^3} dx.$$

(g) Use first derivative test to find the local maxima and local minima if the function $f(x) = x^3 - 12x$. 5

(h) Prove that the three medians of a triangle meet at a point called centroid of the triangle which divides each of the medians in the ratio 2 : 1. 5

2. (a) Verify that $2 + 2^2 + ... + 2^n = 2^{n+1} - 2$, using the principle of mathematical induction. (Here, n represents natural numbers). 5

(b) Determine the 10th term of the Harmonic Progression $\frac{1}{7}, \frac{1}{15}, \frac{1}{23}, ...$. 5

(c) Evaluate : 5

$$\int \frac{dx}{\sqrt{x} + x} .$$

(d) Solve the following system of equations, by using Cramer's rule : 5

$$x + 2y + 2z = 3$$
$$3x - 2y + z = 4$$
$$x + y + z = 2 .$$

3. (a) Given $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$. Verify that $xyz = a^3 + b^3$. 5

(where ω is cube root of unity and $\omega \neq 1$).

(b) Given $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, perform the following : 5+5

(i) Determine A^{-1} and A^3.

(ii) Verify that $A^2 - 4A - 5I_3 = O$.

(c) If the roots of $ax^3 + bx^2 + cx + d = 0$ are in A.P., show that $2b^3 - 9abc + 27a^2d = 0$. 5

4. (a) Determine the points of local extrema of the function : 5

$$f(x) = \frac{3}{4}x^4 - 8x^3 + \frac{45}{2}x^2 + 2015 .$$

(b) Calculate the shortest distance between vectors $\vec{r}_1$ and $\vec{r}_2$ given below : 5

$$\vec{r}_1 = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + \lambda)\hat{k}$$

and

$$\vec{r}_2 = 2(1 + \mu)\hat{i} + (+2 - \mu)\hat{j} + (-1 + 2\mu)\hat{k}$$

(c) Determine the values of x for which the function $f(x) = 5x^{3/2} - 3x^{5/2}$, $(x > 0)$ is : 5

(i) increasing

(ii) decreasing

(d) If $|z - 2i| = |z + 2i|$, verify that $\text{Im}(z) = 0$ (where z is a complex number). 5

5. (a) Find the direction cosines of the line passing through the two points (1, 2, 3) and (–1, 1, 0). 5

(b) Find the area bounded by the curves $y = x^2$ and $y = x$. 5

(c) Two tailors A and B earn ₹ 150 and ₹ 200 per day respectively. Tailor A can stitch 6 shirts and 4 pants while Tailor B can stitch 10 shirts and 4 pansts per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minimum labour cost ? Also calculate the least cost. 10